3. Modalanalyse Rezeplanz, $f=1$

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$$m\ddot{q} + k\dot{q} + cq = \hat{F}\sin(\Omega t) \quad \Big| \div m$$

$$\ddot{q} + 2D\omega_0\dot{q} + \omega_0^2 q = \frac{\hat{F}}{m}\sin(\Omega t)$$

Ansatz: $q_p = \hat{q}\,e^{j(\Omega t+\varphi)}$; $\quad \hat{F}\sin(\Omega t) = \hat{F}e^{j\Omega t}$

$$\hat{q}\left(-\Omega^2 + 2D\omega_0 j\Omega + \omega_0^2\right)e^{j\varphi} = \frac{\hat{F}}{m} \quad \Big| \div \hat{F}\,;\ \div (\,)$$

$$\underline{\frac{\hat{q}}{\hat{F}}e^{j\varphi} = \frac{1}{m}\cdot\frac{1}{-\Omega^2 + 2D\omega_0 j\Omega + \omega_0^2} =: H_R(j\Omega)} \quad \text{Rezeplanz}$$

① $$H_R(\Omega=0) = \frac{1}{m}\cdot\frac{1}{\omega_0^2} = \frac{1}{\cancel{m}\cdot\frac{c}{\cancel{m}}} = \frac{1}{c} = Re\{H_R(\Omega=0)\}$$

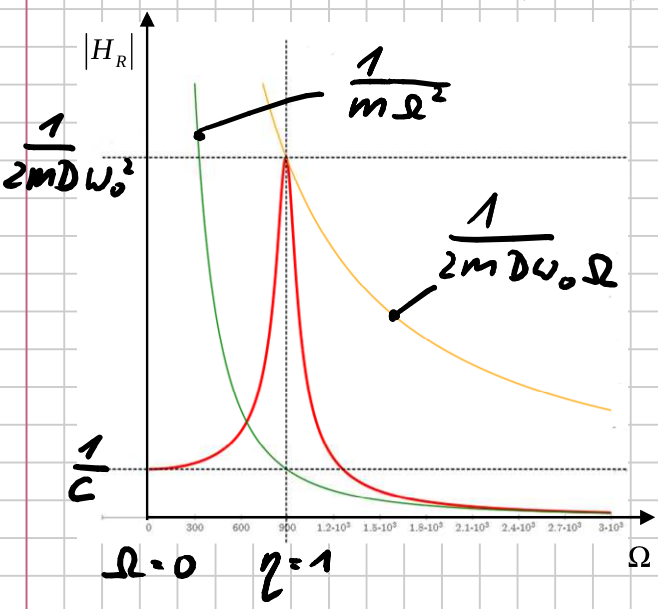
② $$H_R(\Omega=\infty) = \frac{1}{\infty} = 0 = Re\{H_R(\Omega=\infty)\}$$

③ $$H_R(\Omega=\omega_0) = \frac{1}{m}\cdot\frac{1}{-\cancel{\omega_0^2} + 2D\omega_0 j\omega_0 + \cancel{\omega_0^2}} = \frac{1}{j2mD\omega_0^2}\cdot\frac{j}{j}$$

$\eta = 1$

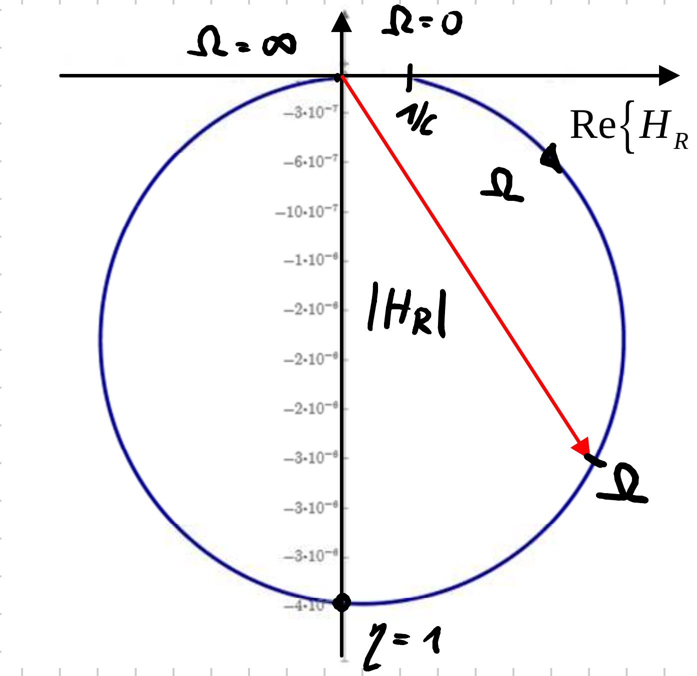
$$H_R(\Omega=\omega_0) = -j\cdot\frac{1}{2mD\omega_0^2} = -Im\{H_R(\Omega=\omega_0)\}$$

Amplitudenfrequenzgang $|H_R(j\Omega)|$



Ortskurve von $H_R(j\Omega)$

$\mathrm{Im}\{H_R(j\Omega)\}$



① $\Omega = 0$

$$C = \frac{1}{\mathrm{Re}\{H_R(\Omega = 0)\}}$$

③ $\zeta = 1$

$$-\mathrm{Im}\{H_R(\Omega = \omega_0)\} = \frac{1}{2 m D \omega_0^2}$$

$$D = \frac{-1}{2 m \omega_0^2 \, \mathrm{Im}\{H_R(\Omega = \omega_0)\}}$$